Maths for Computing
Assignment 1

1. *(5 marks)* Construct a truth table for the following compound propositions.
   a) $p \rightarrow (\neg q \vee r)$
   b) $(p \rightarrow q) \vee (\neg p \rightarrow r)$

2. *(5 marks)* Write each of these statements in the form "if p, then q" in English.
   a) I will remember to send you the address only if you send me an e-mail message.
   b) To be a citizen of this country, it is sufficient that you were born in the United States.
   c) If you keep your textbook, it will be a useful reference in your future courses.
   d) The RedWings will win the Stanley Cup if their goalie plays well.
   e) That you get the job implies that you had the best credentials.

3. *(5 marks)* Show that the following pairs are logically equivalent.
   a) $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$
   b) $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$

4. *(5 marks)* A collection of logical operator is called functionally complete if every compound proposition is logically equivalent to a compound proposition involving only these logical operators. Show that $\neg$ and $\wedge$ form a functionally complete collection of logical operator.

5. *(7.5 (= 2.5 + 5) marks)* Show that the following pairs are logically equivalent without using truth table.
   a) $\neg(p \vee (\neg p \wedge q)) \equiv \neg p \wedge \neg q$
   b) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r) \equiv T$

6. *(5 marks)* Show that the following pairs are not logically equivalent
   a) $\exists x P(x) \rightarrow \exists x Q(x)$ and $\exists x Q(x) \rightarrow \exists x P(x)$
   b) $\forall x P(x) \vee \forall x Q(x)$ and $\forall x (P(x) \vee Q(x))$
   Note that to show that they are not logically equivalent you need to choose predicates and domain for which both WFFs of a pair have opposite truth value. You should choose exactly one domain for a variable that is present in both WFFs.

7. *(3 marks)* Find the truth value of the following propositions where the domain is the set of positive integers. Justify your answer briefly.

a) $\forall x \exists y (x = 1/y)$

b) $\forall x \exists y (y^2 - x < 100)$

c) $\forall x \forall y (x^2 \neq y^3)$

8. *(10.5 (= 3.5 * 3) marks)* Show that the following arguments are valid. (Write all the steps with reasons.)

a) Premises: $p \rightarrow (\neg r \rightarrow \neg q)$, $\neg r$. Conclusion: $\neg(p \land q)$

b) Premises: $(p \lor (q \lor r)) \land (p \leftrightarrow s)$, $q \rightarrow t$, $t \rightarrow \neg q$. Conclusion: $p \lor r$

c) Premises: $\forall x (P(x) \lor Q(x))$, $\forall x ((\neg P(x) \land Q(x)) \rightarrow R(x))$.

Conclusion: $\forall x (\neg R(x) \rightarrow P(x))$ (Domain for all quantifiers are the same.)